# Engineering Notes

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## A Note on the Two Force Equations for a Floating Platform

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TWO approaches in general have been used for the equations of the hydrodynamic forces for a floating, multihull platform in waves such as semisubmersible drilling platform. It is assumed that the submerged portion of the platform members consists of many continuous segments, vertical or horizontal (Fig. 1). The "first" approach[1-6] is to directly use an available method for the wave force and the force due to the segment motion in still water. The "first" approach was initially applied by Bain[1] for the project MOHOLE, and is the same as the method for the wave-induced inertia force plus drag force on a submerged body commonly used in the field of coastal engineering.[7] The "second" approach applies potential flow theory, which has commonly been used for the ship motion problem, and derives the Froude-Krilov force, the difffraction force, and the force due to the segment motion in still water. The second approach was applied by Kim[8] to the floating platform problem without relating to the first approach.

There has been a little confusion on the clear difference and similarity between the two approaches, although the two approaches have been used for the practical applications. The two approaches are compared and are shown to lead to the same equations with certain assumptions. We shall begin with the second approach using the potential flow theory with zero forward speed, used by Salveson et al.[9] for the ship motion problem, and lead it to the forms of the hydrodynamic force equations used by the first approach.

A Cartesian coordinate system $(\bar{x},\bar{y},\bar{z})$ has its origin at the center of gravity of a floating, multihull platform located for convenience at the undisturbed free surface (0,0,0) (Fig. 1). The $x$-axis is positive forward, the $y$-axis is positive toward port, and the $z$-axis is positive upward. Another coordinate system $(x,y,z)$ is introduced with its origin at a point $(x_c,y_c,0)$ of the segment. The present subject is treated in the $(x,y,z)$ coordinate system only.

### The First Approach

The first approach can be expressed in a general form to describe the hydrodynamic forces[5] on the submerged member-segments of a stationary floating platform in waves,

$$X_j^W = (m+a_{jj})\int_L \dot{v}_j \mathrm{d}x_L + b_{jj}\int_L v_j \mathrm{d}x_L \tag{1}$$

which can be divided into two parts:

$$X_j^{F-K} = m\int_L \dot{v}_j \mathrm{d}x_L \tag{2}$$

$$X_j^C = a_{jj}\int_L \dot{v}_j \mathrm{d}x_L + b_{jj}\int_L v_j \mathrm{d}x_L \tag{3}$$

and

$$X_j^M = (a_{jj}\ddot{\zeta}_j + b_{jj}\dot{\zeta}_j)L \tag{4}$$

where all quantities in Eqs. (1-4) are defined for the mass center of a segment, and $m$ is the displaced-water mass by a segment per unit length, $L$ is the segment length, $v_j$ and $\dot{v}_j$ are the water-particle oscillatory velocity and acceleration, respectively, at the mass point in the $j$th direction ($j=1,2,3$), and a dot on top of $v_j$ indicates time derivative. $a_{jj}$ and $b_{jj}$ are the added mass and damping coefficients per unit length, respectively, for the cylinder segment. $x_L$ is a length variable along the segment axis. $\dot{\zeta}_j$ and $\ddot{\zeta}_j$ are the velocity and acceleration, respectively of the $j$th mode of the segment motion.

$X_j^W$ in Eq. (1) is the wave force in a commonly used form which can be divided into $X_j^{F-K}$, the resultant force of a pressure distribution equal to that of the undisturbed waves acting on the segment and $X_j^C$ in Eq. (3), a correction to the $X_j^{F-K}$, to take into account the disturbance of the incident waves by the presence of the segment. $X_j^M$ in Eq. (4) is the force due to the segment motion in still water.

### The Second Approach

For the second approach we shall utilize the work by Salveson et al.[9] for the ship motion with zero forward speed in the derivation of the hydrodynamic forces on the segment. Details of the assumptions and the boundary conditions involved can be referred to that paper.

Let the linearized velocity potential $\Phi(x,y,z;t)$ be

$$\Phi(x,y,z;t) = \phi_T(x,y,z)e^{i\omega_0 t} \tag{5}$$

where $\phi_T$ is the complex amplitude of the unsteady potential and $\omega_0$ is the wave frequency. The amplitude of the unsteady potential is linearly decomposed,

$$\phi_T = \phi_I + \phi_D + \sum_{j=1}^{3} \zeta_j \phi_j^0 \tag{6}$$

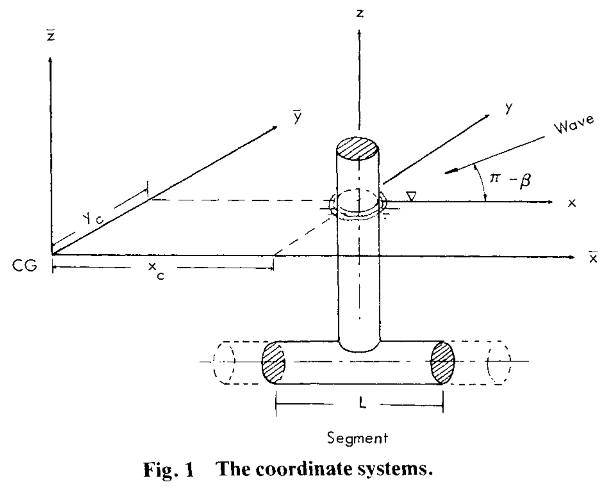


Fig. 1 The coordinate systems.

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Here $\phi_I$ is the incident wave potential. $\phi_D$ is the diffraction potential. $\zeta_j$ are the complex amplitudes of the $j$th mode of motion displacements, and $\phi_j^0$ is the oscillatory potential for the $j$th mode of the segment motion.

The potentials $\phi_I, \phi_D$, and $\phi_j^0$ must satisfy the linear boundary conditions on the segment surface at its mean position

$$(\partial\phi_I/\partial n)+(\partial\phi_D/\partial n)=0 \tag{7}$$

and for the $j$th mode of oscillation

$$\partial\phi_j^0/\partial n=i\omega_0 n_j \tag{8}$$

where $n_j$ $(j=1,2,3)$ are the components of the outward normal vector $\bar{n}$. The potentials $\phi_I$, $\phi_D$, and $\phi_j^0$ must each satisfy additional conditions: the Laplace equation in the fluid domain, the linearized free-surface condition, and the appropriate boundary condition at infinity, and the bottom condition for the case of finite depth.

Integration of the pressure given by the linearized Bernoulli equation[9] over the segment surface and applying Eq. (6) gives the hydrodynamic force amplitudes in three parts as

$$H_j=F_j^I+F_j^D+G_j \tag{9}$$

$F_j^I$ is the amplitude of the Froude-Krilov force, $F_j^D$ is the amplitude of the diffraction force, and $G_j$ is the amplitude of the force due to the segment motion. Indeed $(F_j^I+F_j^D)$ is the exciting force, and will be shown to be the same as the wave force in Eq. (1). These three force components in Eq. (9) are defined as

$$F_j^I=-i\rho\omega_0\int\int_s n_j\phi_I\mathrm{d}s \tag{10}$$

$$F_j^D=-i\rho\omega_0\int\int_s n_j\phi_D\mathrm{d}s \tag{11}$$

$$G_j=-i\rho\omega_0\int\int_s n_j\sum_{k=1}^{3}\zeta_k\phi_k^0\mathrm{d}s$$

$$=\sum_{k=1}^{3}T_{jk}^0\,\zeta_k \tag{12}$$

where $\rho$ is mass density of water.

For a long slender member, $\mathrm{d}s=d\ell\ \mathrm{d}x_L$, $\partial/\partial x\ll\partial/\partial y$ or $\partial/\partial z$ for the horizontal member, and $\partial/\partial z\ll\partial/\partial x$ or $\partial/\partial y$ for the vertical member. $d\ell$ is the element of arc along the cross-section $C_x$. Then the potential $\phi_j^0$ can be reduced to two-dimensional problem of a cylinder: $\phi_k^0=\psi_k$, $k=2,3$ for the horizontal segment and $k=1$, 2 for the vertical segment. Hence,

$$T_{jk}^0=\int_L \mathrm{d}x_L(-i\rho\omega_0\int_{C_x} n_j\psi_k\mathrm{d}\ell)=\int_L t_{jk}\mathrm{d}x_L \tag{13}$$

where $t_{jk}$ can be evaluated as,

$$t_{jk}=\omega_0^2 a_{jk}-i\omega_0 b_{jk},\quad j,k=1,2,3 \tag{14}$$

where $a_{jk}$ and $b_{jk}$ are two-dimensional added-mass coefficients and wave-damping coefficients per unit length of the segment. For the cylinder segment of constant $C_x$ along $x_L$, $t_{jk}=0$ for $j\neq k$, and the diagonal coefficients, $t_{jj}$ $(j=1,2,3)$ are the only nonzero coefficients.

Now we shall transform the forms of Eqs. (10-12) for the hydrodynamic forces to the forms of Eqs. (2-4) used by the first approach. Applying Gauss' theorem to change the surface integral in Eq. (10) to the volume integral gives the Froude-Krilov force,

$$F_j^I e^{i\omega_0 t}=\rho\int\int\int \dot{v}_j\mathrm{d}V \tag{15}$$

where

$$\dot{v}_j=(\partial/\partial t)\ (\partial\phi_I/\partial x_j) \tag{16}$$

$$\phi_I(x,y,z)=(ig\alpha/\omega_0)\exp$$

$$[-ik(x\cos\beta-y\sin\beta)]e^{kz} \tag{17}$$

and $V$ is the segmental volume. Now it is assumed that the cross-sectional dimension of the segment is small compared to the wavelength and water depth, that the force acts on the displaced-water mass centerline of the cylinder segment, and that the wave surface is uniform across the cross-sectional surface of the segment about the waterplane center of the segment. The first assumption implies $ky\ll 1$ and $kz\ll 1$ for the horizontal segment, $kx\ll 1$ and $ky\ll 1$ for the vertical segment, and $kh\geq 0(1)$ where $h$ is the water depth. Then Eq. (15) becomes

$$F_j^I e^{i\omega_0 t}=\rho A\int_L \dot{v}_j\mathrm{d}x_L=m\int_L \dot{v}_j\mathrm{d}x_L \tag{18}$$

The preceding assumptions allow the area, $A$, and displaced-water mass per unit length, $m$, of the cylinder segment to stay outside the integral. References 2, 3, and 5 which treat the finite-depth problem by the first approach, failed to point out the assumption that the cross-sectional dimension of the segment is small compared with the water depth.

The Froude-Krilov force of Eq. (18) obtained by the potential flow theory is exactly the same as Eq. (2) obtained by the first approach

$$F_j^I e^{i\omega_0 t}=X_j^{F-K} \tag{19}$$

Obviously, $X_j^{F-K}$ in Eq. (2) is subject to the assumptions and boundary conditions used for $F_j^I$ in Eq. (18). It is noted that Paulling[3] and Chung[6] used the second approach to get the $X_j^{F-K}$.

Next, applying Green's second identity to the diffraction force, Eq. (11) and applying the boundary condition, Eqs. (7), to it, respectively, give

$$F_j^D=-\rho\int\int_s \phi_j^0\frac{\partial\phi_D}{\partial n}\mathrm{d}s \tag{20}$$

$$=\rho\int\int_s \phi_j^0\frac{\partial\phi_I}{\partial n}\mathrm{d}s \tag{21}$$

where

$$\partial\phi_I/\partial n=(-i\,n_1\cos\beta+i\,n_2\sin\beta+n_3)k\phi_I \tag{22}$$

Applying the assumptions used to get Eq. (18) and the relationship, Eq. (14) to Eq. (21), respectively, give

$$F_j^D=\int_L \mathrm{d}x_L\frac{\partial\phi_I}{\partial x_j}\int_{C_x}\rho n_j\phi_j^0\mathrm{d}\ell \tag{23}$$

$$F_j^D e^{i\omega_0 t}=-\frac{1}{i\omega_0}(\omega_0^2 a_{jj}-i\omega_0 b_{jj})\int_L v_j\mathrm{d}x_L \tag{24}$$

$$=a_{jj}\int_L \dot{v}_j\mathrm{d}x_L+b_{jj}\int v_j\mathrm{d}x_L \tag{25}$$

The coupling coefficients in Eqs. (23-25) are zero as previously described. Usual consideration of the viscous effect on the damping can be empirically incorporated into the wave damping coefficients, $b_{jj}$. The diffraction force of Eq. (25) obtained by the potential flow theory is exactly the same as Eq. (3) obtained by the first approach;

$$F_j^D e^{i\omega_0 t} = X_j^C \tag{26}$$

The force due to the segment motion without applying the assumptions used to get Eq. (18) is obtained by applying the relationship (14) to (12),

$$G_j e^{i\omega_0 t} = \zeta_j (\omega_0^2 a_{jj} - i\omega_0 b_{jj}) L$$
$$= -(a_{jj}\ddot{\zeta}_j + b_{jj}\dot{\zeta}_j) L \tag{27}$$

The coupling coefficients in Eq. (27) are zero as described previously. The force due to the motion of the segment in still water, Eq. (27) obtained by the potential flow theory is exactly the same as Eq. (4) obtained by the first approach

$$G_j e^{i\omega_0 t} = X_j^M \tag{28}$$

In summary, the two approaches give idential equations of the hydrodynamic forces on the platform segment in the $(x,y,z)$ coordinate with the assumptions made to get Eq. (18). Without these assumptions, the second approach[8] is more general than the first approach. However, the computed results by the two approaches for a floating platform in waves in the $(\bar{x},\bar{y},\bar{z})$ coordinate system can be compared with caution. There are the hydrodynamic interferences at the joints of the vertical and horizontal segments and between the adjacent segments. The hydrodynamic interferences at the joint can generate a three-dimensional flowfield. For the three-dimensional hydrodynamic interferences at the joints, the equations of the diffraction force and motion-dependent force should use the added mass and damping coefficients which take into account the hydrodynamic interferences at the joints: if the interferences are strong, theory may not be valid. The first approach, within the framework of the theory[9] can be compared to the second approach with the computed results which take into account the hydrodynamic interferences. For this reason the computed results by Kim[8] can be compared to the first approach only with caution. It is noted that both approaches have been applied to many platform problems with reasonably good accuracy of the computed results.

## References

[1] Bain, J. A., "Extension of MOHOLE Platform Force and Motion Studies," Oct. 1964, Project MOHOLE Report by General Electric Co.

[2] Burke, B. G., "The Analysis of Motions of Semisubmersible Drilling Vessels in Waves," *Proceedings of Offshore Technology Conference*, Houston, Texas, 1969, Paper 1024.

[3] Paulling, J. R., "Elastic Response of Stable Platform Structures to Wave Loading," *Proceedings of the International Symposium on the Dynamics of Marine Vehicles and Structures in Waves*, Institute of Mechanical Engineers, London, England, April 1974, pp. 263-272.

[4] Ochi, M. K. and Vuolo, R. M., "Seakeeping Characteristics of a Multi-Unit Ocean Platform," *Transactions of the Society of Naval Architects and Marine Engineers*, Honolulu, Hawaii, May 1971.

[5] Hooft, J. P., "A Mathematical Method of Determining Hydrodynamically Induced Forces on a Semisubmersible," *Transactions of the Society of Naval Architects and Marine Engineers*, Vol. 79, New York, 1971, pp. 28-70.

[6] Chung, J. S., "Motions of Semisubmersible Drilling Rigs in Deep Water," *Journal of the Society of Naval Architects of Korea*, Vol. 11, Nov. 1974, pp. 23-40.

[7] Wiegel, R. L., *Oceanographical Engineering*, Prentice-Hall, Englewood Cliffs, N. J., 1964, pp. 254.

[8] Kim, C. H. and Chou, F., "Motions of a Semi-Submersible Drilling Platform in Head Seas," OE 71-8, Dec. 1971, Ocean Engineering Department, Stevens Institute of Technology, Hoboken, N. J.

[9] Salveson, N., Tuck, E. O., and Faltinson, O., "Ship Motions and Loads," *Transactions of the Society of Naval Architects and Marine Engineers*, Vol. 78, New York, 1970, pp. 250-287.

# Turbulent Wake of an Axisymmetric, Self-Propelled Body

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## Nomenclature

| | |
|---|---|
| $A,B,C,C_1,C_2,D$ | = constants |
| $\ell$ | = length scale |
| $u'_m$ | = turbulent velocity scale |
| $u'_{max}$ | = measured maximum turbulent intensity at a given axial position |
| $U_0$ | = freestream velocity |
| $-\rho \langle u'_r u'_z \rangle$ | = Reynolds stress |
| $U_d$ | = velocity defect scale |
| $U_z$ | = mean axial velocity |
| $r_{1/2}$ | = measured length scale |
| $r,z$ | = radial, axial coordinate |
| $\epsilon$ | = kinematic eddy viscosity |

## Introduction

THOUGH free turbulent shear flows have been investigated extensively, both analytically and experimentally, comparatively little analytical work has been done in studying the wakes of bodies with hydrodynamic self-propulsion.[1] A very complete experimental study was done, however, in which the wake of a totally immersed, axisymmetric, self-propelled body was simulated in an air tunnel using a concentric nozzle and disk. The results, reported by Naudascher,[2] provide enough data to check the validity of proposed hypotheses of the wake behavior and also the corresponding analytical solutions. One important conclusion from the data was that various flow characteristics, such as mean velocity, turbulence intensity and turbulent Reynolds stress, attained a self-preserving form. Thus, a self-preservation hypothesis seems justified as the basis for an analytical approach to the problem. That is, the flow characteristics may be assumed invariant along the wake axis if expressed in terms of appropriate length and velocity scales. Similarity solutions of this type are given by Birkhoff and Zarantonello,[3] and Tennekes and Lumley[4] in which one length and one velocity scale are used. The results indicate that the length scale should have the form $C_1 z^{1/5}$, while the velocity scale should vary as $C_2 z^{-4/5}$. No analytical expression, however, was given for the corresponding mean velocity profile in the axisymmetric case.

When these classical results are compared with Naudascher's data, plotted in Fig. 1, it is seen that the predicted axial dependence of the length scale is roughly the same as the data. The expression for the velocity scale, on the other hand, agrees well with the observed data for the turbulence intensity, but not with the mean velocity defect. In fact, as pointed out by Schetz and Favin,[1] Naudascher's data show that the turbulence intensity and the velocity defect do not vary axially in the same way, thus distinguishing this case from more conventional wake flows.

In this Note, therefore, a new similarity solution is developed in which two velocity scales are used—one for the mean velocity defect and one for turbulent velocity fluctuations. Expressions for the mean velocity profile as well as the velocity and length scales are developed.

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